



VOLTIMISE VÕLU

Raili Vilt

TÜ teaduskool

Eesti Matemaatika Selts

16. august 2021

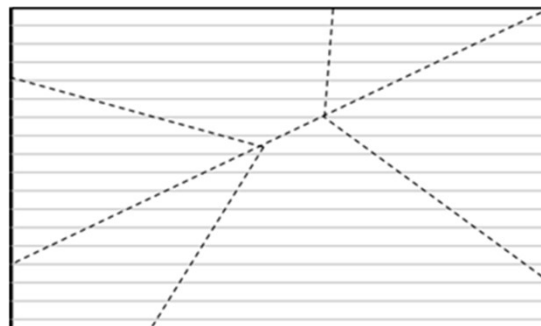
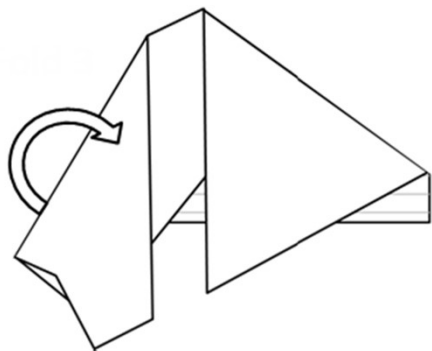
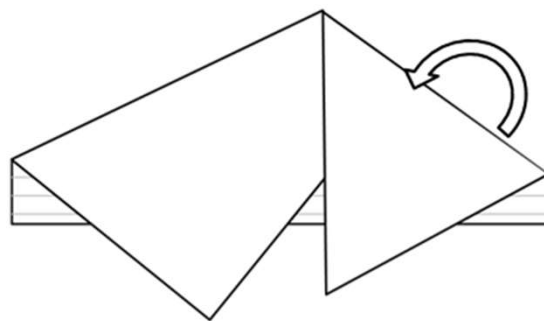
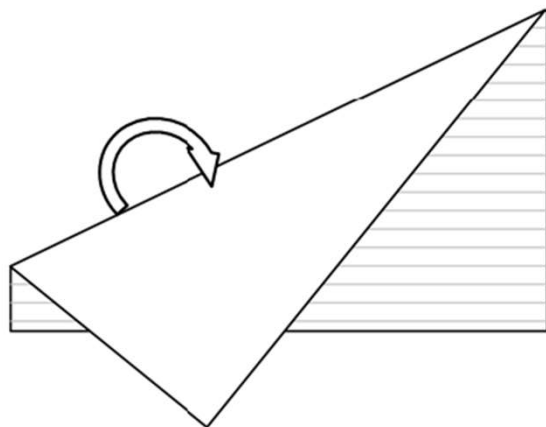
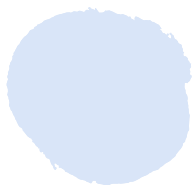


Võlud

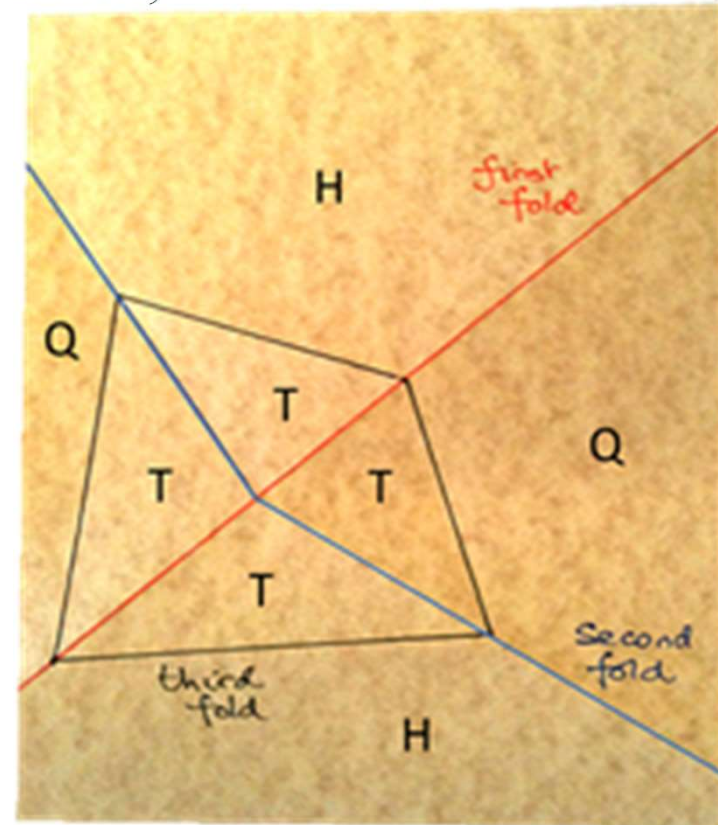


Vähesed vahendid



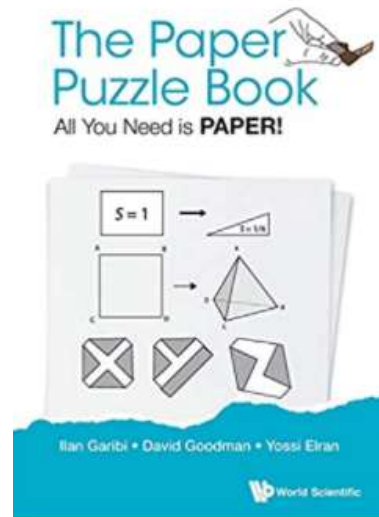


Voltida 3 korda sirgjooneliselt nii,
et tulemuseks oleks
4 kolmnurka,
2 kuusnurka ja
2 nelinurka.



Palju põnevat nuputamist näiteks raamatus

The Paper Puzzle Book (Ilan Garibi, David Hillel Goodman, Yossi Elran)



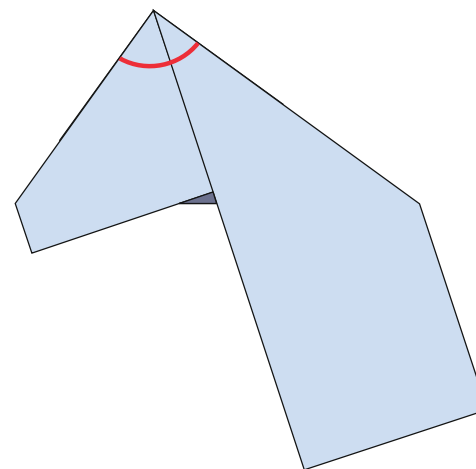
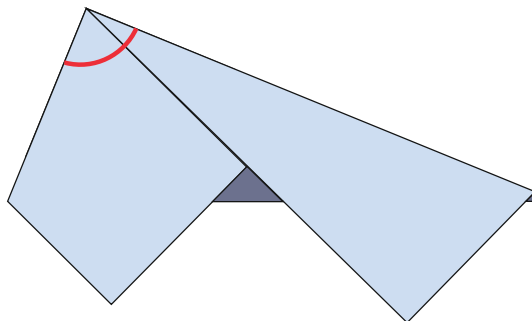
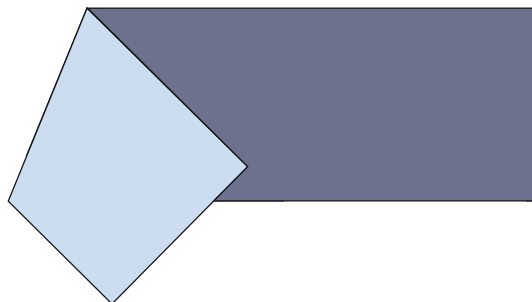


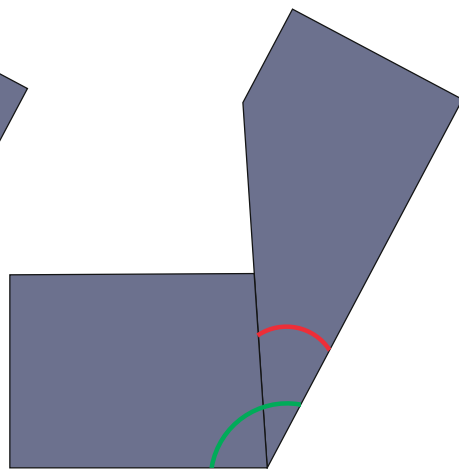
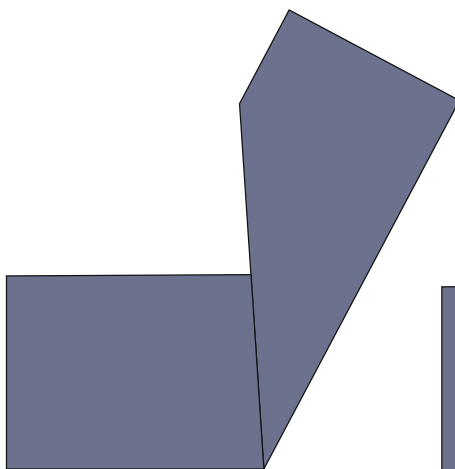
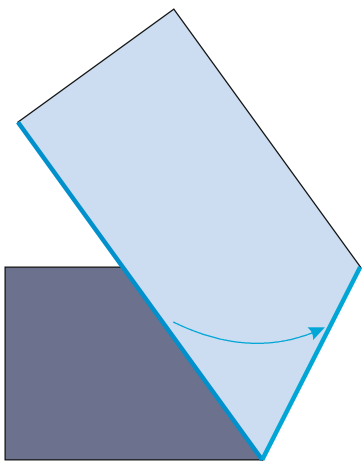
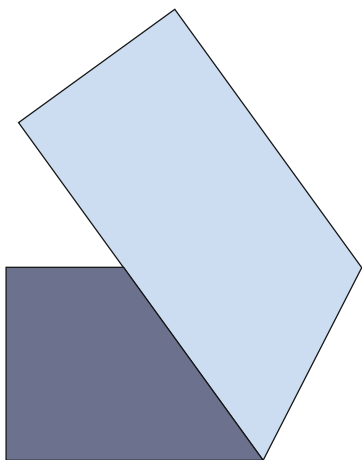
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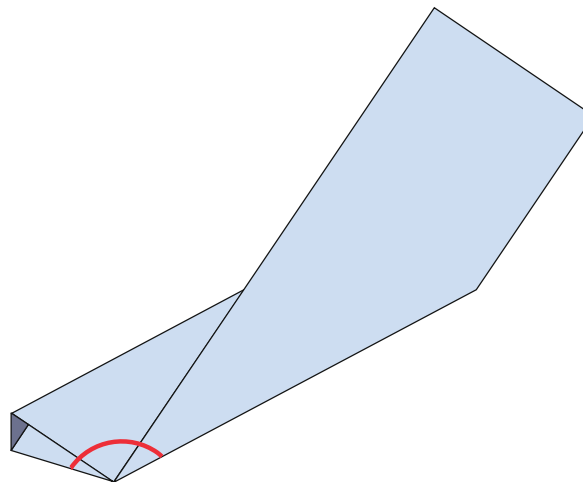
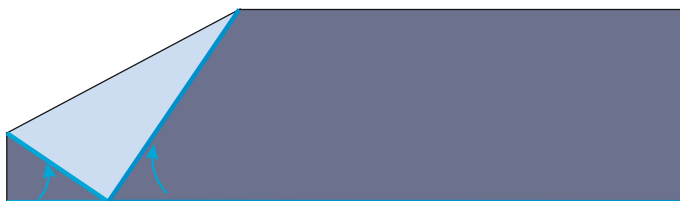
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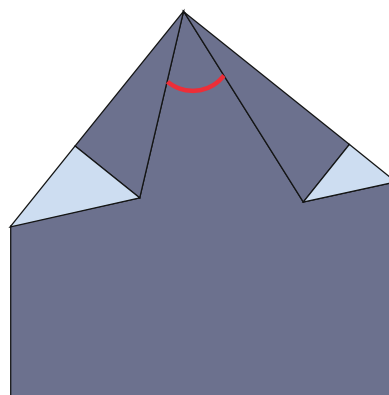
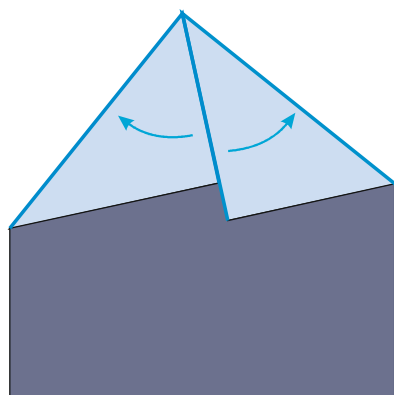
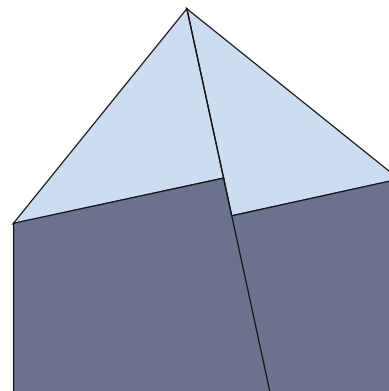
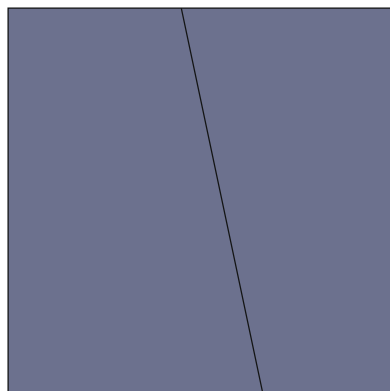
Konstruksioonid, arvutamine

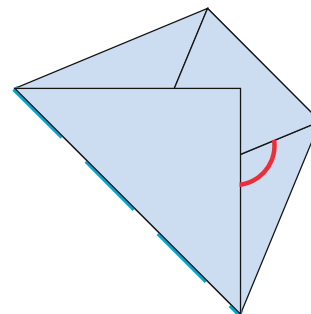
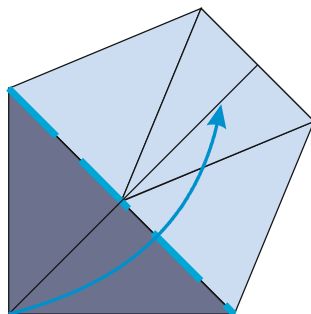
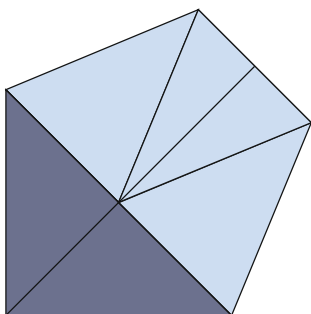
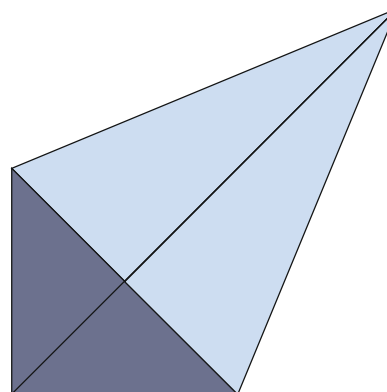
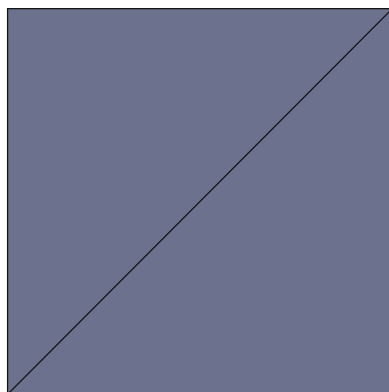
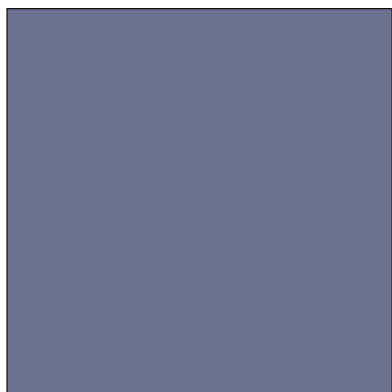






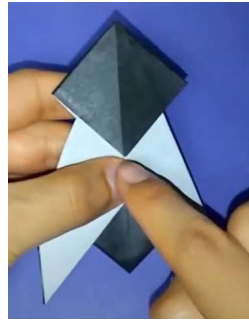
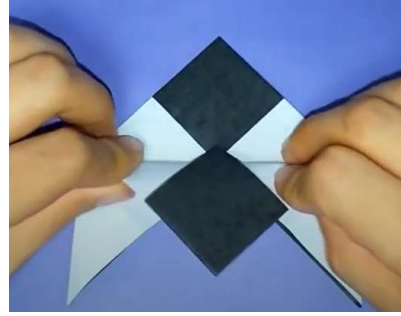
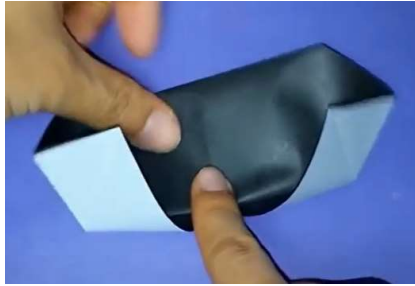
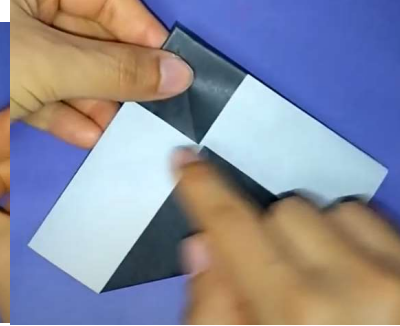
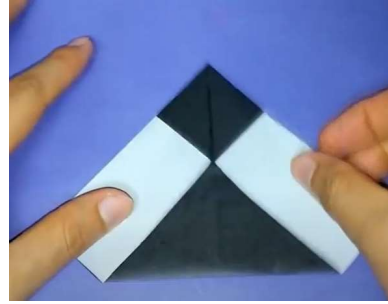
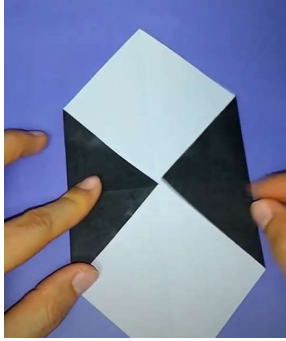


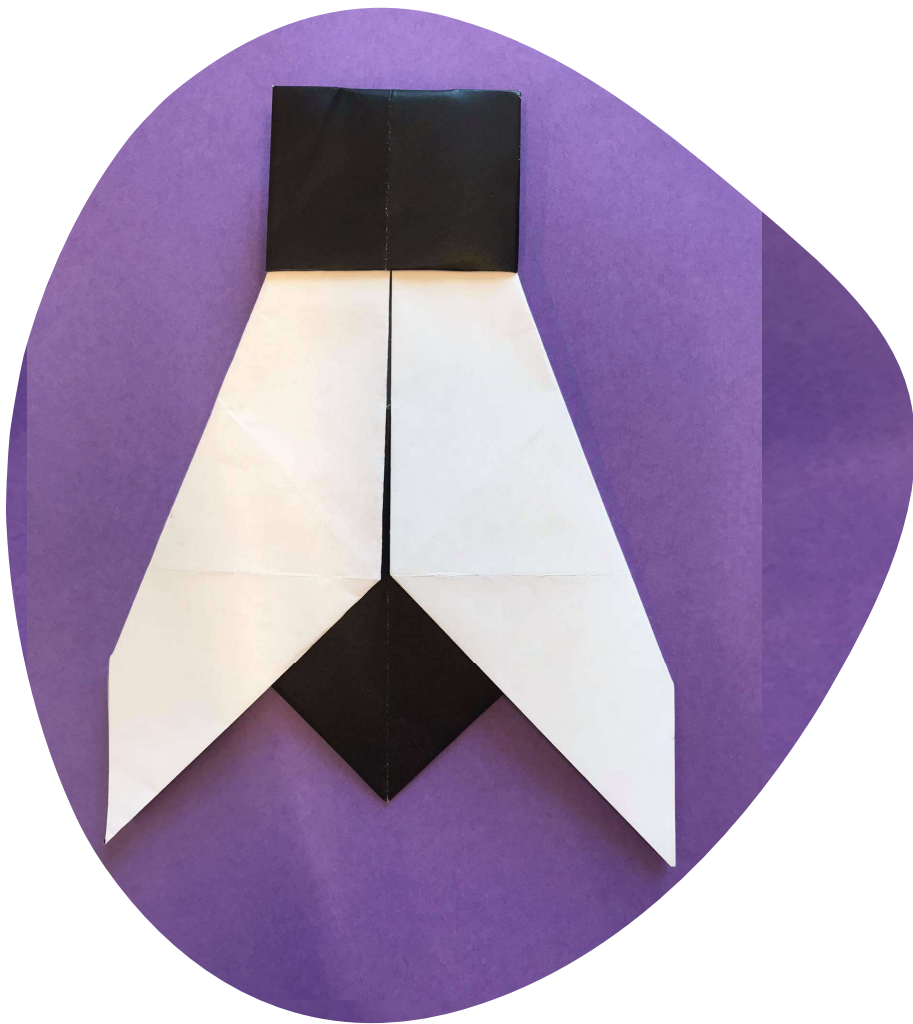


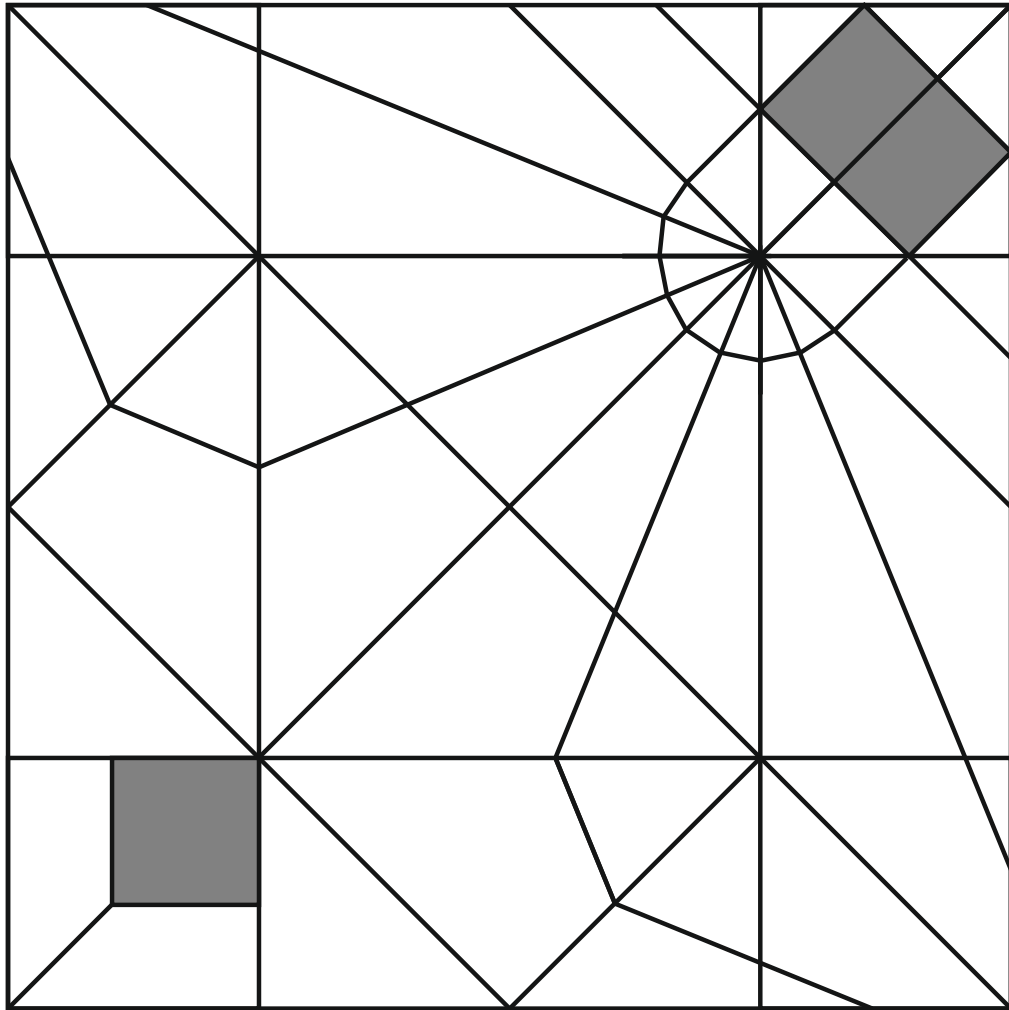


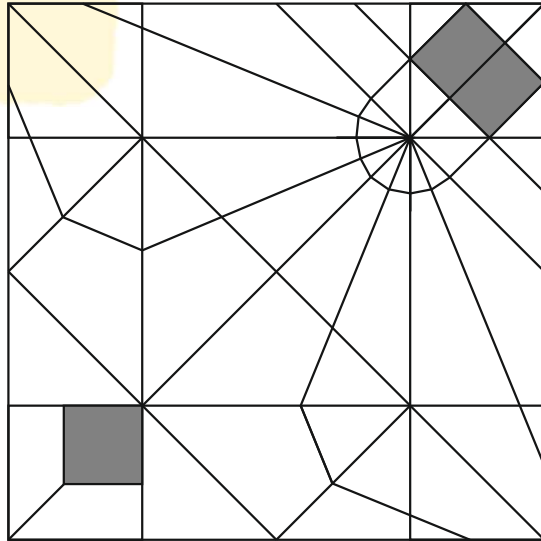


<https://www.youtube.com/watch?v=OmLhCyX9YcE>

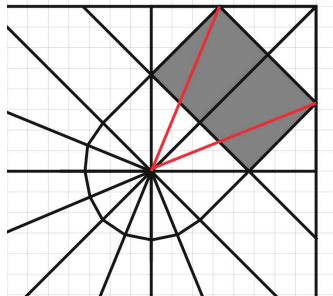








Antud ruudu külg $4a$.



Sabaks jääv must ruut.

Diagonaali pikkus: $2a\sqrt{2} - 2a = 2a(\sqrt{2} - 1)$

Pindala: $4a^2(\sqrt{2} - 1)^2 = 4a^2(3 - 2\sqrt{2})$

Peaks jääv must ristkülik.

Ruut küljega a .

Vaatleme täisnurkset kolmnurka nurkadega 90° , $22,5^\circ$ ja $67,5^\circ$ ning mille pikem kaatet on a ja lühem x .

Seega $\tan 22,5 = \frac{x}{a}$.

$\tan 22,5 = \sqrt{2} - 1$.

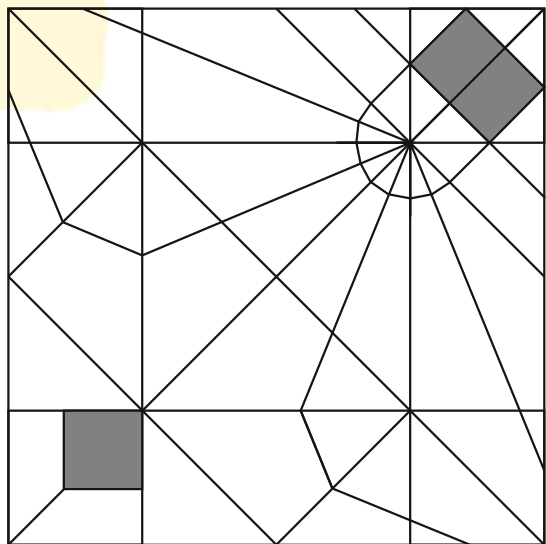
Seega $x = a(\sqrt{2} - 1)$.

Musta ristküliku lühem külg on $a\sqrt{2}(\sqrt{2} - 1) = a(2 - \sqrt{2})$.

Musta ristküliku pikem külg:

$\sqrt{2}(a - x) = \sqrt{2}(a - a(\sqrt{2} - 1)) = \sqrt{2}(2a - a\sqrt{2}) = 2a(\sqrt{2} - 1)$.

Pindala: $a(2 - \sqrt{2}) \cdot 2a(\sqrt{2} - 1) = 2a^2(3\sqrt{2} - 4)$



Antud ruudu külg $4a$.

Musta osa kogupindala

$$a^2(3 - 2\sqrt{2}) + 2a^2(3\sqrt{2} - 4) = a^2(4\sqrt{2} - 5)$$



Võlud



Vähesed vahendid



Konstruksioonid, arvutamine

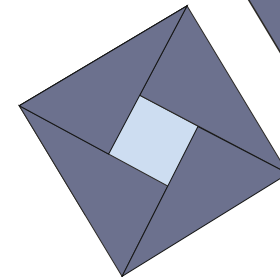
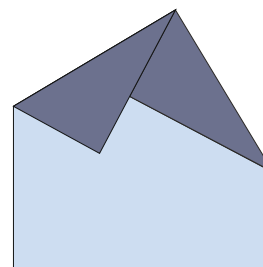
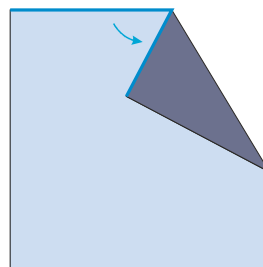
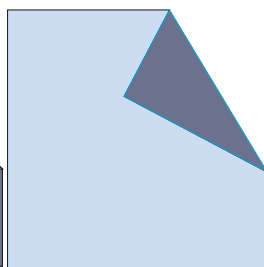
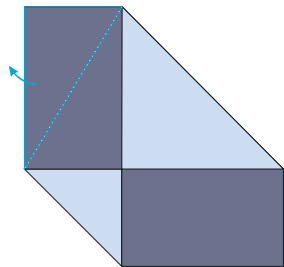
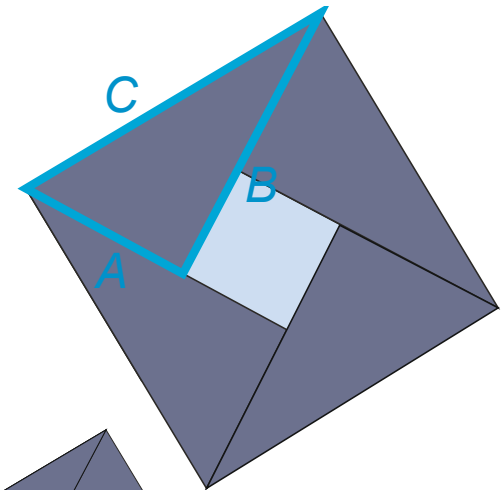
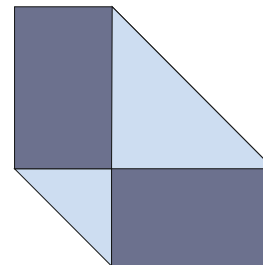
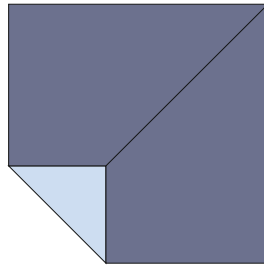
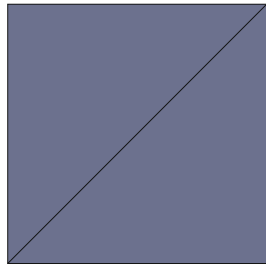
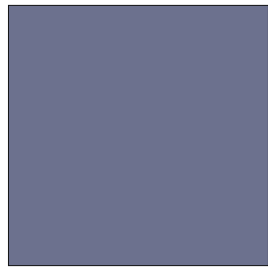


Tõestuste illustreerimine



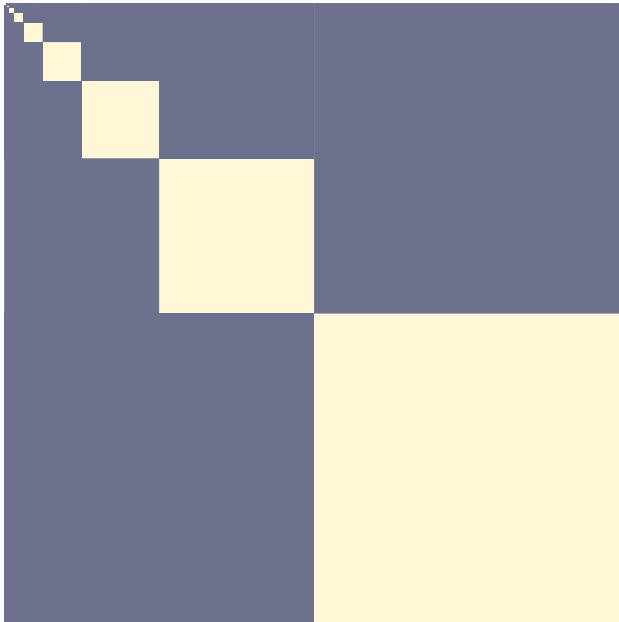
Klassika

$$c^2 = 4 \cdot \frac{A \cdot B}{2} + (B - A)^2$$



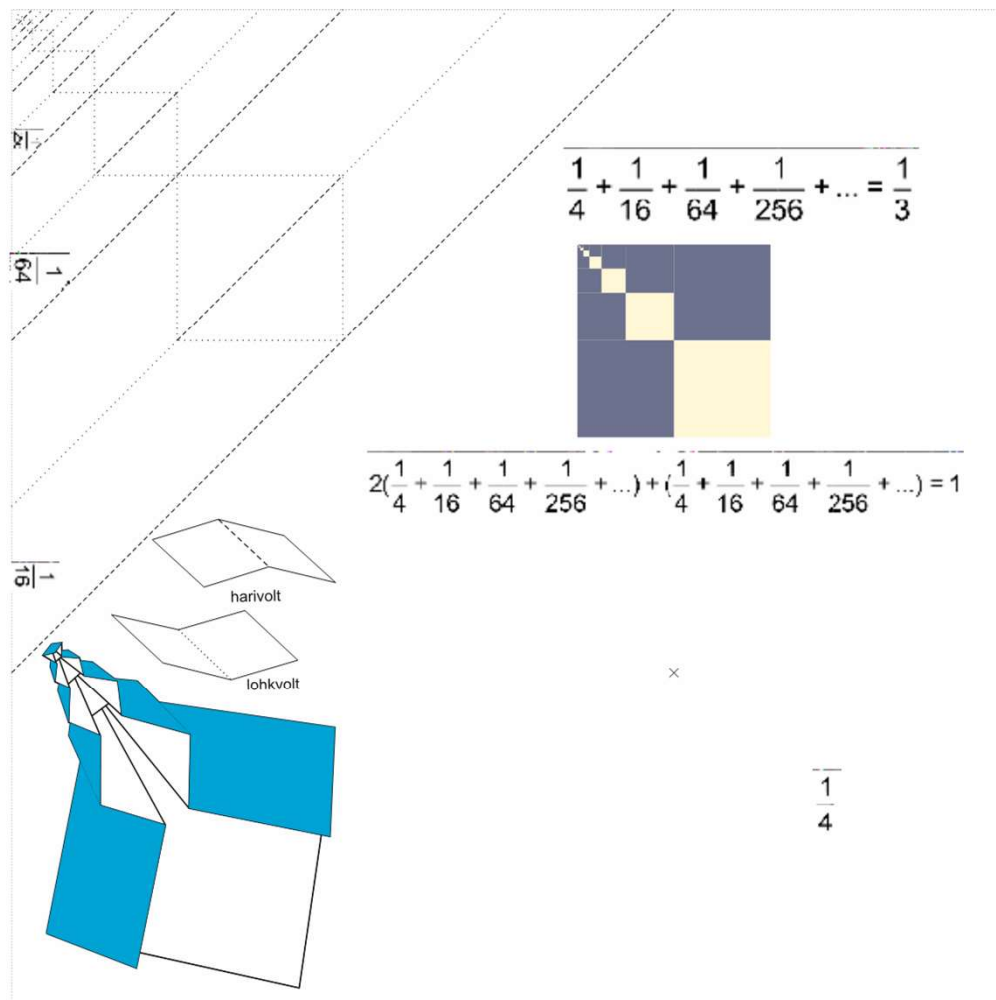


$$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \dots = ?$$



$$2\left(\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \dots\right) + \left(\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \dots\right) = 1$$

$$\frac{1}{4} + \frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \dots = \frac{1}{3}$$



EMS - asutatud 1926, taasisutatud 1987
KMÜ - asutatud EMS allorganisatsioonina 1989

Koolimatemaatika Ühendus

- **kaitseb** ja **esindab** liikmete ühiskondlikke ja kutsealaseid huve
- **loob** võimalusi ja **aitab kaasa** liikmete erialase kvalifikatsiooni tõstmisele
- **edendab** ja **toetab** liikmete kutsealast tegevust
- **teeb koostööd** HTM-i ja HARNO-ga, ülikoolidega ning teiste õpetajate ühendustega nii Eestis kui rahvusvaheliselt
- **aitab kaasa** koolimatemaatika arengule ja matemaatiliste teadmiste levitamisele
- **hoolitseb** matemaatilise hariduse eest kõigis kooliastmetes
- **korraldab** õpilasvõistlusi ja -üritusi
- **edastab** informatsiooni

Astu liikmeks!
Ole aktiivne liige!

Me ❤️ matemaatikat!





Tänan!